

Sketching Graphs Using Transformations

1) What sequence of transformations will change $y = |x|$ into $y = 3|1 - x|$.

* Factor the leading coefficient out of the x term. $3|1 - x| = 3|(-1)(x - 1)| = 3|x - 1|$ (this last step because of the properties of absolute value).

* $(x) \Rightarrow (x - 1)$ is a **transformation**. What would the value of our “new” function be if x were equal to 4? We’d subtract 1 from 4, and see that the value of the “new” function is the **same** as the value of the “original” function at $x = 3$. So – the effect of this **transformation** is to move the original function to the right by 1.

* Once we get *that* y value, we then multiply it by 3. So this has the effect of stretching the function vertically: a y value of 1 becomes 3; a y value of -0.5 becomes -1.5; and so on.

Now graph this and see the effect of these transformations.

Is there a ‘short cut’ to figuring out these transformations?

I don’t know ... but when I think about this, I begin with the changes closest to the x . Remember the general form of a SIN related function: $y = A \sin(B(x + C)) + D$. If you factor the B out like this, things become much clearer....

Working from the x out ... we look at $+C$ first; then the B ; then the A ; and finally the D .

$X + C$ determines the horizontal translation (this was called Phase Shift).

The factor of B determines horizontal stretching (this helped determine Frequency/Period).

The factor of A determines vertical stretching (... Amplitude).

Finally, the $+ D$ is a vertical translation (was this called anything?).

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There’s another way to go about graphing. Look at “Sketching Graphs of Logarithmic Functions”, number 1.

1) $3 \ln(4x) + 1$. Rewrite this as $3 \ln(4(x - 0)) + 1$. So – no horizontal translation. A horizontal dilation (factor of 4). Then a vertical stretching (factor of 3). Then ... up 1.

An alternative way to go about this: Start with your knowledge of $\ln(x)$. When x is 0, $\ln(x)$ has a vertical asymptote. When x is 1, $\ln(x)$ is 0. And when x is e , $\ln(x)$ is 1.

Now ... the technique (which sounds so much better than “trick”).

What makes $(4x) = 0$? An x value of 0 ... that’s where the vertical asymptote is. What makes $(4x) = 1$? An x value of $1/4$. What makes $(4x) = e$? An x value of $e / 4$. That gives us a y value of 1.

After we have those “key” points, we just multiply the y values by 3 to sketch our curve